Number factorization with structured light

Number factorization plays a prominent role in network systems and cyber security, as well as in optimization. It has also become a crucial ingredient of a host of promising physics-based protocols for information encoding, optical encryption, and all-optical machine learning. Lately, intriguing prime number links have emerged to multifractality of light in photonic arrays undergirded by algebraic number theory.

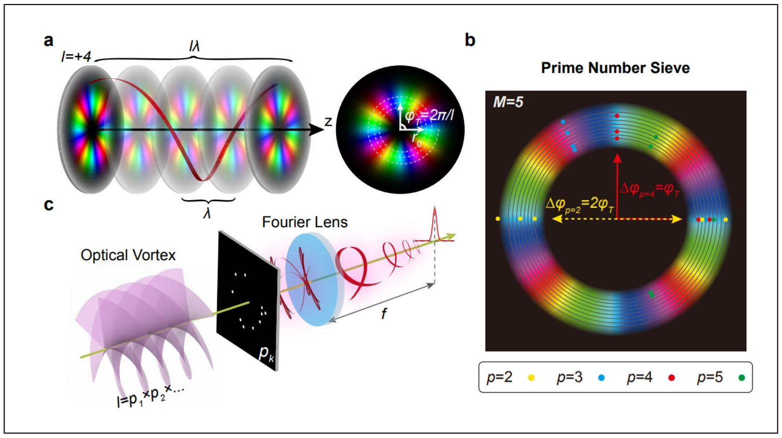

FIG. 1. Prime number factorization with OAM beams. (a) The phase distribution of an OAM beam twisted as a spiral staircase during propagation exhibits azimuthal periodicity with the period $\phi_T = 2\pi/l$. (b) Prime number sieve examples marked with color dots. (c) Implementing prime number factorization with OAM beams; the hue and brightness display phase and intensity.

We have recently established an instructive connection between structured light physics and number theory. Specifically, we derived a remarkably simple analytical relation between the normalized intensity autocorrelation function of structured paraxial random light fields at a pair of points on an optical axis of the system and an incomplete Gauss sum, a crucial tool of number theory for finding prime factors of numbers. The discovered connection enabled us to advance and experimentally implement a protocol to factor large numbers on an optical analog computer [1]. The protocol makes it possible to factor numbers as large as a few million, but it takes long time to process data associated with requisite large statistical ensembles of structured light beams. To help speed up the protocol, we proposed and implemented two alternatives. The first relies on periodic properties of the orbital angular momentum (OAM) of coherent light beams and a carefully prepared optical "prime number sieve" as is illustrated in Fig.1 ; see [2] for further detail, while the second exploits a connection between transverse correlations of the fields of judiciously engineered structured random light beams and incomplete Gauss sums [3]. We have collaborated with the groups of Prof. Cai and my former graduate, Prof. Liang from Shandong Normal University, China where all experiments were carried out.

[1] X. Liu, C. Liang, Y. Cai, and S. A. Ponomarenko, "Axial correlation revivals and number factorization with structured random waves," Phys. Rev. Applied, **20**, L021004 (2023).

[2] X. Li, X. Liu, Q. Wu, J. Zeng, Y. Cai, S. A. Ponomarenko, and C. Liang, "Prime number factorization with light beams carrying orbital angular momentum," APL Photon. **9**, 046107 (2024).

[3] T. Gao, X. Liu, Q. Chen, S. A. Ponomarenko, Y. Cai, and C. Liang, "Prime number factorization and degree of coherence of speckled light beams," Opt. Lett. **49**, 5232 (2024).